SECTION I.—AEROLOGY.

THE INFLUENCE OF METEOROLOGICAL CONDITIONS ON THE PROPAGATION OF SOUND.

By HARRY BATEMAN, Ph. D. Johnston Scholar.

(Dated Johns Hopkins University, Baltimore, Md., June 5, 1914.)

§1. THE INFLUENCE OF THE WIND.

The effect of wind on the velocity of propagation of sound was first established by a long series of experiments by Dr. Derham (1). He observed that the time taken by the report of a cannon in traveling a distance of 12½ miles across the Thames from the grounds of Blackheath varied between 55½ and 61 seconds according as the wind and sound traveled in the same or in opposite directions. A negative result had previously been obtained by some members of the Accademia del Cimento at Florence, but the observations were made at distances of not more than two or three miles from the source of sound and the difference in the time due to the effect of the wind is so small that, as Derham remarked, it is not surprising that it escaped observation.

Derham paid more attention to the effect of wind on the velocity of sound than to its effect on the range and audibility of sound. A passage which Derham quotes from an account of the work of Joseph Averiani (2) shows that the existence of the latter effect was recognized, but its cause was not then understood.

> Nor is there need of favoring winds to promote this passage of sound in order that it may be surely heard. Indeed, any wind whatsoever, whether it be favorable or adverse, is equally an impediment, and renders the sound less audible. It may be because the roar of the sea, agitated by this cause, is more a disadvantage than the current of air blowing in the same direction is an advantage. Hence it is that the sound is heard only when the wind is entirely still or is only murmuring very gently, when the air is serene and the sea tranquil. Nor then, indeed, is it heard indiscriminately from all points, but from those only which are a little the more elevated * * *.

These remarks are interesting because two phenomena are recorded which have been observed in more recent experiments. The effect of wind in reducing the audibility of sound was noticed, for instance, by de la Roche (3) who compared the sounds from two equal bells placed at such distances from the observer in different directions that the sounds from them appeared to be equally distinct. De la Roche found that the distance for a direction at right angles to the wind was greater than for directions with or against the wind. This result led some men to the erroneous conclusion that sound had a greater *range* in a direction at right angles to the wind than in other directions. This error was corrected by the experiments of Joseph Henry (4) and Osborne Reynolds (5), which showed very clearly that the range with the wind is generally about double that at right angles to the wind. Reynolds says:

> It does not follow, however, nor was the fact observed that at comparatively short distances the sound with the wind was more *intense* than at right angles.

De la Roche, Henry, and Reynolds also noticed that a sound which was practically inaudible close to the ground or surface of the sea could often be heard at a greater altitude, for instance on the top of a tree or cliff or at the masthead of a ship. Thus Averiani's observation has been abundantly confirmed. The well-known fact that the sound of a bell can be heard over a wider area when the bell is raised above the ground belongs to the same order of ideas. This fact has been applied with advantage in the case of school bells and church bells.

These phenomena seemed very mysterious until it was shown by Stokes and Reynolds that they were due to refraction of sound waves by the atmosphere. At the Dublin meeting of the British Association in 1857 Stokes pointed out that when the velocity of the wind increases overhead rays of sound traveling to windward are gradually bent upward and at a moderate distance pass over the head of the observer while rays traveling with the wind are bent downward. An observer to leeward of the source hears by a direct ray which starts with a slight upward inclination and has the advantage of being out of the way of obstruction for the greater part of its course (6). Stokes explained De la Roche's observation that at short distances sound is most intense in a direction at right angles to the ground by saying that in this direction a ray which reaches the observer after being reflected from the ground has very nearly the same direction as the direct ray, consequently the two rays produce a greater intensity of sound in this direction than in any other.

This idea of a refraction of sound due to a vertical gradient of wind velocity was adopted and developed by Joseph Henry in his discussion of the results of the experiments with fog signals made by a committee appointed by the United States Lighthouse Board. (7)

The same idea was evolved by Reynolds before he became acquainted with the work of Stokes, and was used to explain the results of his experiments. It was also taken up by Alexander Beazeley (8), who suggested that the range of sounds to be sent against the wind could be increased by raising the source and projecting the sounds slightly downward.

The fact that the propagation of sound can not be described so accurately as in the case of light, by using the idea of a ray has been regarded as a difficulty in Stokes's theory. Reynolds discusses the question by considering the analogous case of diffraction of light and concludes that at short distances the effect produced by a lateral diffusion of the sound would be small. Major Dutton (9) considered that the lateral diffusion might to some extent counteract the effect of refraction and Henry used the same idea to explain how sound which had been refracted upward could reach the earth again (10). Henry also showed that a return of the sound to the earth could be accounted for in another way, by postulating the existence of an upper current of air blowing in the opposite direction to the lower one. The latter bends the rays of sound upward when they are directed against it, whereas the former current causes a downward bend which brings the rays to earth again. With the aid of this idea Henry accounted for the regions of silence and abnormal audibility observed by Gen. Duane in 1871 during a northeast snowstorm (11). The idea of an upper current blowing in the opposite direction to the lower one has been described by Tyndall and other writers as an "invention" or "conjecture," but in justice to Henry it should be mentioned that he regarded the upper current as a condition naturally associated with a northeast storm on the east coast of this country. Moreover, in September, 1874, he ascertained by means of toy balloons that the upper current was blowing in the direction he anticipated and was in the direction of the maximum sound range.

Tyndall also criticized Henry's theory by saying that he did not understand how the sound wave could recross the hostile lower current. (Preface to Tyndall's Sound, 3d edition, pp. 19–20.) To meet this objection Mr. W. B. Taylor (**12**) has drawn a figure to illustrate the action of a compound wind in changing the direction of the sound rays, and this seems to make the matter clear.

Lord Rayleigh (**13**) has studied the reflection and refraction of sound at a horizontal surface, in crossing which the velocity of the wind changes discontinuously; it appears that if the upper velocity is the greater some of the rays will be totally reflected. He also remarks that sound moving to leeward over still water, being confined between parallel reflecting planes, diverges in two dimensions only and may therefore be heard at distances far greater than would otherwise be possible. Another possible effect of the reflector overhead is to render sounds audible which in still air would be intercepted by hills or other objects intervening.

When there is a strong upper current blowing in the opposite direction to the lower one, the preceding remarks are applicable to the case of sounds projected against the surface wind whether the bending downward of the sound by the upper current is supposed to be due to reflection or refraction.

In the mathematical theory of the refraction of sound in a windy atmosphere it is necessary to distinguish between the direction of a ray and that of the wave normal. The direction of the ray may be obtained geometrically as follows (**14**): Let P' be the position to which a point P would be carried in time ∂t if it moved with the velocity of the wind. Describe a sphere of radius $V\partial t$ round each such point P' where V is the local velocity of sound, and let Q be the point in which the sphere associated with P is touched by the envelope of all the spheres associated with the different points of the wave front. Then PQ is the direction of the ray at P. It follows from this construction that the ray-velocity is the vector sum of the wind-velocity and the local velocity of sound along the normal to the wave front.

This law can be obtained analytically by regarding the wave fronts as characteristics and the rays as bicharacteristics (**15**) of the partial differential equations of wave propagation in a windy atmosphere.

When the wind is blowing in one direction only, Fujiwhara (**16**) has shown that if viscosity, conduction of heat, and changes of composition are neglected, the sound motion can be derived from a velocity potential ϕ which satisfies the partial differential equation

$$\left(\frac{\partial}{\partial t}+u_1(z)\frac{\partial}{\partial x}\right)^2\phi=c^2\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}+\frac{1}{\rho_1}\frac{\partial\rho_1}{\partial z}\frac{\partial\phi}{\partial z}\right)$$

The wind velocity $u_1(z)$ is parallel to the axis of x and is assumed to depend only on the height z above the earth's surface; ρ_1 is the mean density of the air at height z and c is the local velocity of sound.

Fujiwhara has endeavored to give a more physical meaning to the rays by regarding them as curves along which the energy of the sound motion can be supposed to flow. To reconcile this definition with the previous definitions of the rays he uses a particular solution of the partial differential equation and determines the flow of energy for the corresponding type of wave propagation. This particular solution is of the form

$$\phi=\psi(z)e^{\alpha}$$

where $\alpha\equiv iqt-lx-my+\int ndz$ is a solution of the differential equation of the characteristics, and q, l, m, are constants. With the aid of this expression for ϕ he verifies the above-mentioned law for the ray velocity and then proceeds to apply this law to determine the paths of the rays in the case of sound waves issuing from a point source.

This method is not rigorous, but the results which are obtained are fairly satisfactory, especially as they indicate the existence of a silent region and an area of abnormal audibility in the case when the wind velocity first increases and then decreases as z increases.

For our present purpose (**17**) it will be sufficient to consider the path of a ray which is in the plane of xz. The law for the ray velocity then gives

$$x=\int_0^z\frac{u_1(z)+cl}{cn}dz$$

where l, m, n, are the direction cosines of the wave normal. Let v be the actual velocity of the wave in the direction of the normal, then it follows from the geometrical method or from the differential equation of the characteristics that

$$v=lu_1+c.$$

Now the velocity of the trace of the wave front on a horizontal plane at which there is a change of wind velocity remains constant as we cross the plane, hence the law of refraction states that

$$\frac{v}{l}\equiv u_1+\frac{c}{l}\qquad\text{is a constant }\lambda.$$

Putting $m=0$, $n=\sqrt{1-l^2}$, $l=\frac{c}{\lambda-u_1}$, we find (**18**) for the path of a ray the equation

$$x=\int_0^z\frac{c^2+\lambda u_1-u_1^2}{c\sqrt{(\lambda-u_1)^2-c^2}}dz$$

When u_1 first increases with z and then decreases, a ray moving against the wind has a point of inflexion where $\frac{du_1}{dz}$ vanishes. To find the position of the vertex of the ray in this case we use a to denote $u_1(0)$ and b to denote the maximum value of u_1. Then for a ray moving against the wind the initial value of l is negative, and so if a is small, λ is also negative. The ray is symmetrical on the two sides of the vertex, consequently if R is the range on a horizontal plane through the starting point we have

$$R=2\int_a^b\frac{c^2+\lambda u_1-u_1^2}{c\sqrt{(\lambda-u_1)^2-c^2}}\frac{dz}{du_1}du_1$$
$$+2\int_b^{c+\lambda}\frac{c^2+\lambda u_1-u_1^2}{c\sqrt{(\lambda-u_1)^2-c^2}}\frac{dz}{du_1}du_1$$

where both du_1 and $\frac{dz}{du_1}$ are negative in the second integral.

A point on the ground can be reached once, twice, or not at all, according to the number of real roots which this equation in λ possesses.

To get a case in which the integrations can be carried out Fujiwhara assumes that

$$\begin{aligned} u_1(z) &= k_1(z-h)+b \qquad z<h \\ &= k_2(h-z)+b \qquad z>h \end{aligned}$$

and shows that with certain values of the constants there is a minimum value of R and consequently a region of silence. For values of R a little greater than this minimum value there are two rays and consequently the sound may be heard twice. The case in which

$$u_1(z) = b - k(h-z)^2 \qquad b > kh^2$$

might be discussed with advantage, for in this case there is an upper current blowing in the opposite direction to the lower one. The integration can be effected with the aid of elliptic functions. An interesting mathematical problem which has not yet been solved is that of finding a distribution of wind velocity which will bring the rays which travel with the wind in a vertical plane xz to a focus on the ground.

The preceding theory is entirely geometrical and does not give any satisfactory information with regard to the intensity of the sound in the region of audibility. A rigorous mathematical investigation of the propagation of waves sent out from a point source can be made with the aid of the so-called fundamental solution of the partial differential equation. The existence of this solution has been established by E. Holmgren (**19**) and J. Hadamard (**20**). The solution may be derived from the corresponding solution for the ordinary equation of wave motion by a method of successive approximations.

§2. THE INFLUENCE OF INEQUALITIES OF TEMPERATURE.

Derham remarks in his "De Motu Soni":

> I have often observed in summer time, when the air has grown hot, that sounds appeared more languid than usual and were exceedingly weak in their impression on the ear; while in weather of another sort, especially in winter, if it happens to be freezing cold, the same sounds were much more piercing and shrill, and struck the ear more forcibly. Also, when the north or south east wind was blowing, however adversely, I have observed the sounds to be clearer and shriller than if the wind was blowing from contrary quarters, as Kircher also observed at Rome.

It has also been known for a long time that the sound of a distant waterfall can be heard more distinctly at nighttime than during the day. This phenomenon was explained by Humboldt as being due to the presence of inequalities of temperature during the daytime and their absence at nighttime (**21**). He supposed in fact that the inequalities of temperature produced inequalities of density, and that there were partial echoes at the limiting surfaces of rare and dense air.

This explanation is somewhat misleading, because in a gas of constant composition the velocity of sound is independent of the density, but varies as the square root of the absolute temperature (**22**). What is probably the true explanation was given by Osborne Reynolds, who showed that sound waves are refracted in passing through layers of air at different *temperatures*. Total reflection may of course occur in special cases.

If the temperature decreases upward, as it generally does during the daytime, sound rays are refracted upward, and an observer at some distance from the source may be left in a kind of sound shadow. A decrease of temperature of 1 degree Fahrenheit in 100 feet is sufficient to make a difference of about 1 foot a second in the velocity of sound and to produce a curvature equal to that of a circle 104,500 feet in radius. That the temperature gradient may be sufficient to produce quite an appreciable curvature of the rays is illustrated by an observation of Lord Rayleigh on a very hot day when the sound of a passing railroad train was almost inaudible at a distance of 150 yards (**23**). This curvature of the rays makes it possible for sounds to be heard at considerable distances above the earth when they are inaudible at a comparatively small horizontal distance. For instance, in their balloon ascents Flammarion and J. Glaisher have heard the sound of a train at heights of 8,200 and 22,000 feet, respectively (**24**).

At nighttime when the ground has become cool the temperature gradient is generally much smaller and may indeed be inverted owing to the presence of dew, consequently the audibility of sound is reduced very little by refraction.

In a letter to Nature, November 25, 1875 (**13, p. 67**), Dr. Schuster remarks:

> With regard to the question whether our better hearing at night is due to the absence of disturbing noises or to the cause suggested by Prof. Reynolds, I wish to remark that the Upper Himalayas are particularly free from any disturbing noises, yet the increase in our power of hearing at night is most marked.

Richard Townley wrote to Derham in 1704 to say that sounds are rarely heard as far at Rome as in England. Derham, wishing to test this point, caused an inquiry to be made in Italy. Some observations conducted by Joseph Averiani pointed to the conclusion that sounds can be heard as far in Italy as in England, but unfortunately the observations were made at nighttime while Townley's observations were most probably, judging from Derham's account, made in the daytime, and the direction of the wind was unrecorded. It is quite likely that the existence of a large temperature gradient in the latter case was the cause of the discrepancy.

The equation for the path of a ray when the refraction is due only to the temperature gradient is comparatively simple (**25**). The ray evidently lies in a vertical plane; and if θ is the angle which it makes with the vertical, then $c \operatorname{cosec} \theta$ is unaltered by refraction. Hence

$$x = \int_0^z \frac{c\,dz}{\sqrt{a^2 - c^2}}$$

where a is a constant for each ray. An interesting mathematical problem is that of finding a distribution of temperature such as will bring the rays in a vertical plane to a focus. If R is the constant range and h the height of a vertex of a ray we must have $a = c(h)$ and

$$R = 2\int_0^h \frac{c(z)dz}{\sqrt{[c(h)]^2 - [c(z)]^2}}$$

for all values of h. This equation is satisfied by $c(z) = A\cosh\frac{\pi z}{R}$ where A is a constant (**26**). This law would correspond to an increase of temperature with z and could only hold for a short range of values of z.

A useful rule for comparing the refractive powers of wind and temperature is that a difference of 1 foot per second in wind velocity is nearly equivalent to a difference in temperature of 1° F. (**27**).

§3. THE INFLUENCE OF MOISTURE AND VARIATIONS IN THE COMPOSITION OF THE AIR.

The idea that sounds are reflected from clouds was developed by Tyndall (**28**), who supposed that invisible clouds, or a flocculent condition of the air, was produced by streams of water vapor rising from the sea. The aerial or ocean echoes discovered by Henry (29) were attributed by Tyndall to the presence of these invisible clouds, and other abnormal effects were explained as being due to the acoustic opacity of the atmosphere. Tyndall supported his theory by a number of interesting experiments made in the laboratory, but observations which have been made under actual conditions do not seem to favor his views. For instance, in a balloon ascent Flammarion noticed that when the balloon was in the midst of a cloud the sound of a band increased in intensity and the band seemed quite near (30). Again, when Henry tried to obtain an echo from a visible cloud the effect was imperceptible (31).

Henry does not seem to have been satisfied with Tyndall's explanation of the silent region and made some experiments to see if the sound of a fog signal could be intercepted by an acoustic cloud. He used the method of reciprocal observations and showed that sometimes a sound produced at A might be heard at B, while a similar sound produced at B could not be heard at A. Such a failure of reciprocity is probably due to the wind (32).

Henry suggests as a possible explanation of the echo that in the spreading out of the sound from the siren some of the rays take such curved paths that they are reflected back from the sea. A peculiarity of the echo is that it seems to be produced only by signals in which a direction is given to the sound and appears to come from a point on the horizon in the horizontal projection of this direction (33). When Henry ordered the siren to be fixed so that the sound was directed vertically upward the echo appeared to come from the whole of the horizon. It is still thought that no satisfactory explanation of the echo has been given (34).

When a train of waves in a heterogeneous medium is produced by a local disturbance which lasts for a finite interval of time a portion of the medium does not generally remain undisturbed after the wave has passed over but a residual disturbance, which may be of an oscillatory character, is left behind. It is possible that the first maximum of this residual disturbance may correspond to the aerial echo. If this residual disturbance is due entirely to reflections caused by the inequalities of the medium this explanation is practically the same as Tyndall's, but there may be a residual disturbance when the medium is isotropic, as, for instance, when the propagation takes place in two dimensions (35). The propagation of sound through a medium of varying properties has been studied with special reference to the question of reflection by Lord Rayleigh (36) and J. W. Nicholson (37); the latter concludes that Tyndall's explanation of his own experiments is a very possible one. The propagation of sound in a heterogeneous nonabsorbing fluid has also been discussed recently by M. Brillouin (38), who considers the question of dispersion and the variation of amplitude. He shows also that the flow of potential energy does not remain constant along a tube of orthogonal trajectories of the wave fronts (39).

Nicholson says:

> Tyndall's acoustic clouds were regarded by him as being mainly due to the presence of an excess of aqueous vapor in some parts of the atmosphere. Now, moist air has a greater power of radiating heat than dry air, and the consequent "stifling" of the sound passing through very moist air (40) may be appreciable, although in air under ordinary conditions the effect is negligible.
>
> Tyndall does not give a definite idea of the nature of the action of an acoustic cloud, which may act, for the purposes of his theory, by stifling the sound, by scattering, or by reflecting it back to its starting point. Probably the first two effects both play their proper parts. Moreover, as we have shown, the backward reflection when sound enters such a dissipative medium tends to lose its periodicity, and to be independent of the reflecting medium when the dissipation exceeds a certain limit.

The following observation mentioned by Pasquay (41) is of some interest. A whistle was heard at a place 12½ kilometers away from the source of sound and separated from it by two ridges of hills. As this occurred just before rain set in the phenomenon was attributed to the reflection of the waves of sound at the upper layers of air which was saturated with moisture. An upper current of wind or an unusual distribution of temperature might also have produced this effect.

A refraction of sound may also be produced when the percentage of water vapor in the atmosphere varies with the height; for, as Reynolds pointed out, an increase in the amount of water vapor lowers the density and so increases the velocity of sound. An excess of water vapor overhead would produce a downward refraction of the rays, but the curvature would be very small.

H. Mohn (42) and W. R. Livermore (43) have discussed the refraction of sound under the combined action of wind, temperature, and humidity. There is some uncertainty, however, about the exact effect of humidity because it is not known how the ratio of the specific heats varies with the amount of water vapor present in the air. The vapor of γ for dry air is about 1.4 and for water vapor about 1.3. Now Capstick (44) has given a formula

$$\frac{P}{\Gamma-1}=\sum\frac{p_s}{\gamma_s-1}, \qquad \mathrm{P}=\sum p_s$$

for determining the ratio Γ of the specific heats of a mixture of gases when the ratios γ_s and the partial pressures p_s of the components are known. At present it is unknown whether this formula is applicable to a mixture of air and water vapor. Some experiments ought to be made to decide this point.

To account for the areas of silence and abnormal audibility observed at the time of some recent dynamite and volcanic explosions, G. v. d. Borne (45) has introduced the idea of an effect due to the presence in the upper atmosphere of an isothermal region and of air composed mainly of hydrogen. By using Dalton's law for mixed gases, he has calculated the velocity of sound at different heights and determined the paths of the rays. His computed range of the silent regions agrees very nearly with the observations. Borne also considers that there may be a total reflection of rays at a lower boundary of the "hydrogen atmosphere." It is unlikely, however, that there is anything like a sharp discontinuity in the density of the air (46).

Quervain (47) and Fujiwhara (48) are of the opinion that the abnormal range of the sound produced by the dynamite explosion on the Jungfrau railway was due chiefly to the wind and not to the presence of the hydrogen atmosphere. Fujiwhara has made an elaborate study of the sound phenomena associated with the eruptions of Mount Asama and the diagrams in his memoir indicate very clearly the action of the wind (49). Abnormal phenomena relating to the audition of sound have been noticed during the eruptions of Cotopaxi and Krakatoa. There has been some difference of opinion as to the effect of fog on the propagation of sound. Derham remarks:

> A like uncertainty obtains with regard to clear and foggy air. In rainy and damp weather I have often observed that sounds are blunted and that after torrential rains they acquire the greatest strength, as Kircher observed at Rome. But the contrary also often happens. * * * But as regards *thick fogs*, it is certain that they are dampers of sound in the highest degree. For sounds then seem to be for the most part very weak and blunted—a fact which very certainly proceeds

from the interposed vapors and thick particles which compose fog. I have likewise observed the same concerning snowy weather. For when fresh snow has fallen on the ground sounds straightway grow dull; but when its surface has been covered with ice, the sounds suddenly become more acute, and I then have heard bells ringing and cannon booming just the same as if there was no snow on the ground.

This statement of Derham's seems to have given rise to the idea that fog is always prejudicial to the propagation of sound. Although this was denied by Desor (50), the idea was generally accepted until the time when systematic experiments were made with fog signals. Thus Reynolds says (51):

That sound does not readily penetrate a fog is a matter of common observation. The bells and horns of ships are not heard so far during a fog as when the air is clear. In a London fog the noise of the wheels is much diminished, so that they seem to be at a distance when they are really close by. On one occasion, during the launch of the *Great Eastern*, the fog was reported so dense that the workmen could neither see nor hear.

The experiments of Henry, Duane, and Tyndall have shown conclusively that sound can generally be heard quite distinctly in a fog; indeed, the passage of sound through a fog is usually favored by the homogeneous condition of the atmosphere which is the usual concomitant of foggy weather. Some kind of reflection may, however, take place when sound waves pass from clear air into a fog, while if the sound is produced in the fog, its intensity may be greater than in clear air owing to the greater density of the fog.

In a description of some remarkable noises, T. McKenny Hughes (52) remarks that the propagation of the sound is affected by fog. He says that the noises are probably produced in some way by the tide at Morecambe Bay, and compares them with the "barisàl guns." In this case, however, it may be that the sea is comparatively calm when there is a fog, so that the latter hinders the production of the sound rather than its propagation. The propagation of sound through a foggy atmosphere has recently been studied mathematically with the aid of the hydrodynamical equations for the motion of a viscous fluid. C. J. T. Sewell (53) treats the drops of water as minute spherical obstacles and assumes that the volume occupied by the suspended drops is small compared with the total volume of the fog. The first calculation was made on the hypothesis that the drops of water do not move. Sewell then found that if the diameter of each drop is about 0.02 mm., and there are 10^3 drops per cubic centimeter, then the fog does not interfere appreciably with the propagation of sound, but if the diameter of each drop is about 0.002 mm., a fog of the same density would contain 10^6 drops per cubic centimeter and the sound would be damped very quickly by the fog. In a second calculation, made at the suggestion of Prof. Larmor, the drops were supposed to be capable of vibrating with the surrounding air. It was then found that a fog in which the drops have diameters as small as 0.002 mm. should not interfere appreciably with the propagation of sound. Sewell also concluded that when the wave length of the sound is very great, or when the obstacles are extremely minute, a drop vibrates with the surrounding air.

These calculations were made on the assumption that the sound was produced continuously by the source. In the case of sounds of short duration the results may be slightly different.

Duhem (54) has studied the propagation of such sounds through a viscous medium and has shown that a wave of discontinuity, in the sense in which the term is usually used by French writers, can not be propagated at all. To show that disturbances can be propagated, however, Duhem has considered a class of waves which he calls "quasi-waves." At the front of such a wave at least one of the partial derivatives $\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x} \cdots \frac{\partial w}{\partial x} \cdots\right)$ of the component velocities u, v, w, is supposed to undergo a rapid change in crossing a layer of small thickness ε, so that if $\frac{\partial u}{\partial x}$ is one of the derivatives in question the difference between the values of $\frac{\partial u}{\partial x}$ on the two boundaries of the layer is a finite quantity which is not small. Duhem then shows that a "quasi-wave" can only be propagated in a viscous fluid when the thickness ε of the thin layer of transition is at least of the same order of magnitude as the coefficient of viscosity μ. When μ is very small the laws of propagation of these quasi-waves are practically the same as those of waves in a perfect fluid. The case in which the medium is homogeneous has been studied more fully by Le Roy (55) who has obtained a general solution of the partial differential equation

$$\left(\lambda\frac{\partial}{\partial t}+a^2\right)\Delta\phi-\frac{\partial^2\phi}{\partial t^2}=0$$

on which the motion depends. He finds by considering a particular case that the phenomena of propagation are sufficiently clear at the start but that some time after the production of the disturbance the thickness of the quasi-wave increases and becomes more and more badly defined, so that the phenomenon of propagation ends by becoming very vague. He concludes also that in the general case when there is initially both displacement and velocity, it is chiefly the disturbance due to the velocity that causes the flow of energy. In a rigorous discussion of the propagation of sound, the conduction of heat ought also to be taken into account for its effect is of the same order of magnitude as that of viscosity (56).

FOG SIGNALS.

Derham suggests at the end of his memoir that gunshots might be used to enable sailors to ascertain by means of the sound the distance of a ship from another ship or from the land. He does not, however, suggest that the signals should be used in a fog, probably on account of his belief that the sound would be rapidly stifled. The idea that fog tends to intercept or modify sound in its passage through the air led to the consideration whether water itself might not be employed as a medium for the transmission of fog signals, and in 1851 Charles Babbage recommended that some experiments should be made (57). Submarine bells are now used for signaling on many ships. In 1854 some experiments on different means of producing sounds for the purposes of fog signals were made by the engineers of the French lighthouse department, and in 1856 and 1863 similar experiments were started in England and the United States (58).

The extensive observations that were carried out by the United States Lighthouse Board to determine the penetrating powers of the different fog signals under various atmospheric conditions have revealed the existence of some very curious phenomena which have been observed over and over again in subsequent experiments (59).

Chief among these are the alternate regions of silence and audibility discovered by Gen. Duane. Joseph Henry attributed these to the presence of an upper wind blowing in the opposite direction to the lower one, but this can not always be the cause of the phenomenon because regions of silence and abnormal audibility have been noticed in apparently clear weather and at no great distance from the source. The areas of silence around a given fog signal are generally constant in their general position but variable in actual position (60). It seems almost impossible to predict beforehand whether the signal will be heard at a given place or not, and many accidents have occurred at sea owing to the inaudibility of a fog signal which was being sounded as usual.

Tyndall (61) and F. E. Fowle (62) have suggested that the regions of silence and abnormal audibility are due to the interference of the direct rays of sound with those reflected from the surface of the sea or ground, but Lord Rayleigh (63) does not seem to be satisfied with this explanation. Indeed, if the silence were due to interference, it should be possible to recover the sound by ascending the mast of a ship, and the position of a silent region should depend on the pitch of the sound.

The principal fog signals that have been used are the siren trumpet (64), an improved form of siren invented by Mr. Adolphus Brown, of New York, the Daboll trumpet or reed horn invented by Mr. C. L. Daboll and improved by Prof. Holmes, steam whistles, guns, bells, and gongs. Experiments have indicated that the siren is the most powerful and effective signal; the reed horn, although inferior in power, is suitable for situations of secondary importance. At first the siren was blown by steam, but compressed air was found to give better results, probably on account of its greater density (65).

A cylindrical form of siren invented by Slight is now generally used (66), but in Canada a modification of the siren, called the diaphone, is widely employed. Besides the instruments already mentioned there is the whistling buoy, invented by Mr. Courtenay, and explosive signals, such as guncotton. A whistling buoy is often placed in regions where the fog signal has not been heard. Sometimes a double horn or siren is used so as to cover a greater range and avoid the occurrence of silent regions. Lord Rayleigh has suggested that better results might be obtained by using a trumpet with an elliptical mouth (67). This device was tried with success in some experiments that were made in 1901 at St. Catherines Lighthouse in the Isle of Wight (68). The Clyde Navigation Trust has quite recently made some experiments with fog guns charged with gas and an apparatus for firing the guns by means of electric waves. It is thought that such guns can be placed with advantage on dangerous rocks and that the wireless system will remove difficulties which have hitherto been insurmountable.

In spite of the excellence of fog signals disasters occur frequently at sea, and the recent wrecks of the *Titanic* and *Empress of Ireland* point to the conclusion that greater precautions ought to be taken. Sometimes a fog signal can be heard distinctly, but it is difficult to tell from which direction the sound is coming. Mr. Della Torre, of Baltimore, has invented an instrument for overcoming this difficulty and believes that it can be used successfully to locate an iceberg by means of an echo.

Other methods of detecting the presence of an obstacle are sometimes available; for instance, it is well known that many fogs are near the surface of the ocean, and a sailor at the masthead as a lookout is all that is needed. The approach of an iceberg can often be detected by a change in the temperature of the water, and Prof. H. T. Barnes, of Montreal, has devised a sensitive instrument to record such a change.

THE SOUNDS PRODUCED BY BOLIDES OR METEORS.

The noises heard during the fall of a meteor in the Lower Pyrenees, on September 7, 1868, are described by Flammarion as follows (69):

> The disappearance of the meteor was preceded by an explosion. This was followed by a continuous noise like the distant rolling of thunder, then by three or four detonations of extreme violence, which were heard at points distant 50 miles from each other. Immediately after these detonations the inhabitants of Sanguis-St. Étienne heard a hissing noise, like that made by red-hot iron when it is plunged into water, then a dull sound, indicating the fall of a solid body to the ground.

On Christmas Eve, 1873, a remarkable meteor was seen from Washington, and the phenomena relating to it were investigated by Cleveland Abbe (70) and some other members of a special committee. Again there was an explosion followed by a rumbling. The reporters considered the noises to be due not to a single definite explosion of the meteor, but to the concentration at the observer's ear of the vast volume of sound emanating

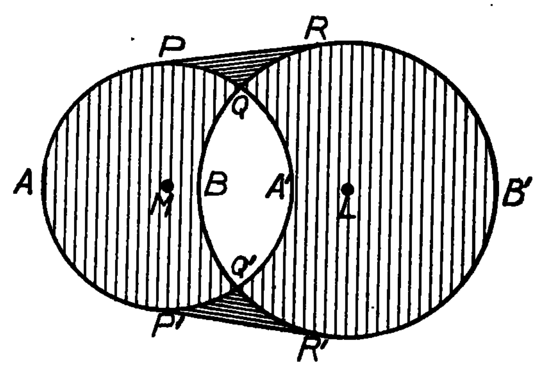

FIG. 1.—Regions about a meteor path.

almost simultaneously from a large part of the meteor's path, being in that respect not dissimilar to ordinary thunder.

This meteor probably described its entire visible path of about 120 miles in three or five seconds; consequently it traveled with a velocity much greater than that of sound. Now, when a body moves with a very great velocity and produces sound for a short interval of time, the elementary spherical waves that issue from its various positions where the sound is produced, have at any instant a real envelope and the disturbance is of a different nature in different regions of space. This may be illustrated by figure 1.

The disturbance which is supposed to be due to the sounds emitted by the friction of the meteor against the air while it describes the path LM is confined within the shaded region. Within the regions PQR, P'Q'R' the sound is concentrated; there are, in fact, two elementary spherical waves through each point. Within the regions PAP'Q'BQ, QRB'R'Q'A', the disturbance differs very little from an ordinary wave of sound, but there is a difference in pitch in the two cases, owing to the Doppler effect. The region of concentration is confined to the neighborhood of a belt or truncated portion of a cone of revolution having LM as axis. This belt is the front surface of a sound bore. At a point of this belt sounds issuing from two consecutive points of the meteor's path reinforce one another. At points of the region of concentration not at the front of the bore the sound is due to elementary waves issuing from different points of the meteor's path, and these may or may not interfere (**71**).

E. Mach thinks that the sound bore that is produced by a projectile traveling with a velocity greater than that of sound is responsible for the explosive sounds that are sometimes heard when a great meteor travels through the air. As I understand it, Abbe's theory of 1877 is an attempt to explain how the sound bore is produced.

The bore produced by a projectile moving with a very high velocity through the ordinary air has been photographed by Mach and much more successfully by C. V. Boys (**72**). It appears that a bullet traveling at a speed of 2,000 feet per second is accompanied by two quasi-conical wave sheets having the same axis. The angle of the cone is such that the velocity of the bullet resolved normal to the wave surface is equal to the velocity of sound in the gas in the condition as to pressure, density, etc., which obtains at the position at which the normal is drawn. The curvature of the generating lines of the surfaces indicates that the condition of the surrounding air is changed for a distance which is considerable in comparison with the dimensions of the bullet (**73**).

In an account of the sound produced by the motion of a bullet Mr. A. Mallock says (**74**):

> Soon after the introduction of modern rifles, which give their projectiles a velocity much higher than that of sound, I noticed that when standing in a position in front of the gun and not far from the line of fire, the sound seemed to come, not from the firing point, but from some point considerably in advance of the gun. The natural explanation seemed to be that the sound thus heard was not that of the explosion itself, but was caused by the wave surface which is generated in the air by the projectile, moving at a velocity higher than sound. It is clear (if the source of sound is due to the wave caused by the projectile) that the apparent direction of the sound will be the normal to the wave-surface, and that if the direction of this normal is known, the velocity of the projectile, at the time that that particular portion of the wave-surface was generated which ultimately reaches the observer, can be calculated.

In the case of a meteor a direct application of this principle is beset with difficulties, owing to the refraction of the sound waves by the wind and temperature gradient.

The Editor having suggested that I should write this review, tells me that Dr. P. J. Nutting, formerly of the United States Bureau of Standards, has assisted him in devising a meteor-recorder by means of which the exact time, location, and velocity of a meteor may be determined. A number of systematic records obtained with the aid of an instrument of this kind would be of great help in clearing up some of the difficulties connected with the sounds produced by great meteors.

Descriptions of the sounds heard by different observers after the fall of a meteor, have been collected on several occasions (**75**) and give a useful qualitative description of some interesting phenomena, but it is desirable that quantitative methods should be used as far as possible.

It is difficult to explain the repeated detonations that are sometimes heard, by the simple bore theory. It is quite likely that echoes arise in some way, either by a total reflection of sound by the wind or a layer of air saturated with moisture or from the unknown cause to which the aerial echo is due.

The fact that the sounds are heard so distinctly, although the meteor is at a great distance from the observer, is attributed by Reynolds to the upward curvature of sound rays due to the refraction by wind and temperature. Observations in balloons point to the correctness of this theory, as sounds are heard very distinctly at a considerable distance from the earth.

NOTES AND REFERENCES.

(1) **William Derham** (1657–1735), English theologian and man of science, was pastor of Upminster in the county of Essex. An account of his researches was published in Latin (Phil. tr., R. soc., London, 1708, **26**). A copy of an English translation by Rev. Dr. J. C. Welling is in the library of the U. S. Weather Bureau. Dr. Welling has also given a brief account of Derham's work in a paper published in the Bulletin of the Philosophical Society of Washington, 1883, **5**, p. 39.

(2) **Averiani** was a professor at the University of Pisa. He had been asked by the Grand Duke to make some experiments that would furnish answers to some questions which had been raised by Derham.

(3) Annales de chimie, 1816, **1**, p. 177.

(4) **Henry, Joseph.** Researches in sound. Smithsonian rept., 1878. This volumes contains a number of diagrams showing the ranges in different directions.

(5) Proc., Royal soc., London. 1874, **22**, p. 531.

(6) Report of the British Association for advancement of science, 1874: Math. and phys. papers, **4**, p. 110. The above condensed version of Stokes's theory is taken from Rayleigh's "Sound," v. **2**, p. 132. Stokes also considers the form of the wave front. This will be discussed more fully when we come to the mathematical theory.

(7) The reports on these experiments date from 1852 when the Lighthouse Board was organized. Henry's reports of 1874, 1875, 1877, and Maj. W. R. Livermore's report of 1894, are of special interest. Henry's experiments were commenced in 1865. They are summarized in Smithsonian report, 1878, and in "Scientific writings of Joseph Henry," **1**, p. 364–510.

(8) *See* **Beazeley, Alexander,** "On coast fog signals," a lecture delivered at the Royal United Service Institution in 1872.

(9) Bull., Phil. soc., Washington, 1875, **2**, p. 59.

(10) **Henry's** Researches, p. 550.

(11) **Henry's** Researches, p. 494. In this case the surface wind was blowing a gale toward the fog whistle and the sound was heard further against the wind than in other directions. An account of some abnormal phenomena of sound noticed during the experiments with fog signals was given by Henry at a meeting of the Philosophical Society of Washington in 1872. Professor Tyndall was present at that meeting. Shortly afterwards Tyndall described some remarkable phenomena he had noticed during his own researches. *See* Bull., Phil. soc., Washington, 1872.

(12) Amer. jour. arts and sci., New Haven, 1876.
Henry's Researches, p. 513.
Livermore's report, 1894.

(13) **Rayleigh.** Theory of sound. **2**, p. 133. The theory is corrected and explained more fully in **Lamb,** "Dynamical theory of sound," p. 220.

(14) **Lamb, H.** Dynamical theory of sound. London, 1910. p. 220.

(15) **Hadamard,** Propagation des ondes. Paris. 1903.

(16) **Fujiwhara** *in* Bull., Central meteorol. obs'y., Japan, Tokyo, 1912, **2**, no. 1. Proc., Tokyo math.-phys. soc., (2) **6**, no. 9, p. 132.

(17) Fujiwhara obtains equations for the path of a ray that starts in any direction.

(18) **Fujiwhara** *in* Jour. meteorol. soc. Japan, Tokyo, 1911, **30**, no. 8, p. 195.
Fujiwhara *in* Bull., Central meteorol. obsy., Japan, Tokyo, 1912, **2**, no. 1, p. 47.

(19) **Holmgren, E.,** *in* Arkiv for matematik och fysik, 1903, 1.

(20) **Hadamard, J.,** in Annales, École normale, (3) 1904, **21**, p. 535.

(21) **Tyndall,** Lectures on sound. 2d ed. 1869. p. 18–19.

(22) Provided the temperature is not high enough to affect the ratio of the specific heats.

(23) Proc. Royal soc., **A86**, 1911, p. 207.

(24) **Flammarion, C.** The atmosphere. (English trans.) London, 1873. p. 80.

(25) **Rayleigh.** Sound. **2**, p. 131.
Lamb. Dynamical theory of sound. London, 1910. p. 217.

(26) The equation is an integral equation of Abel's type and may be solved by a well-known method.

(27) *See* **Livermore,** Report to the Lighthouse Board.

(28) **Tyndall,** Phil. trans., 1874; Sound, 3d ed., chap. 7.

(29) **Henry, Joseph.** Researches in sound. pp. 516, 553, 558.

(30) **Flammarion, C.** The Atmosphere. London, 1873. p. 81.

(31) **Henry, Joseph.** Researches in sound. p. 530.

(32) Tyndall quotes a famous case of a failure in reciprocity when the wind was slight. (Sound, 3d ed., p. 432.) This case has been discussed by Reynolds in Phil. trans., 1876, and in Scientific papers, **1**, p. 157.

(33) *See* Henry and Reynolds, loc. cit.

(34) *See* for example the article "Lighthouses" in Encyclopedia Britannica.

(35) **Lamb, Horace.** Hydrodynamics. 2d ed. Cambridge. 1895. pp. 282, 502.

(36) **Rayleigh.** Theory of sound. **2**, p. 78; Proc. Roy. soc., 1911, **A86**, p. 207.

(37) **Nicholson, J. W.** Proc. Royal soc., 1908. **A81.**

(38) **Brillouin.** Comptes rendus, Paris, 1913, Dec. 8.

(39) In the case of propagation through a windy atmosphere I have not succeeded in proving that the flow of energy at the front of a wave is in the direction of the ray. The rate of change of momentum seems to be in this direction.

(40) **Stokes** *in* Phil. mag., London, April 1851. "Math. & phys. papers," v. 3, p. 142.

(41) **Pasquay** *in* Prometheus, 1903, **14**, p. 384.

(42) **Mohn, H.,** *in* Annalen d. Hydrographie, etc., 1892, 1893, 1895.

(43) **Livermore, W. R.** Report to the Lighthouse Board.

(44) **Capstick,** *in* Phil. trans., Royal soc., London, 1894, A., p. 1.

(45) **von der Borne, G.,** *in* Physikal. Ztschr., 1910, p. 483.

(46) **Humphreys, W. J.,** *in* Bull. Mt. Weather obsy., Washington, 1911, 4.

(47) **Quervain,** *in* Annalen, Schweiz. meteorol. Zentralanst., 1908.

(48) **Fujiwhara,** *see* (*16*).

(49) The conditions prevailing during the eruption of Mount Asama on January 6, 1911, and of Bandaisan correspond to those postulated by Henry. *See* first reference under (*16*), p. 41–44.

(50) **Desor,** *in* Fortschritte der Physik, 1855, **11**, p. 217.

(51) **Reynolds,** *in* Proc. Manchester lit. & phil. soc., 1873–4; Reynolds, Scientific papers, **1**, p. 43.

(52) **Hughes, T. McKenny,** *in* Nature, London, 1895, Nov. 14. Many other remarkable noises are described in letters to "Nature" in answer to a request by Sir George Darwin (Nature, London, 1895, Oct. 31) for information regarding the "barisâl guns" and "mistpoeffers." The former are described in the Proceedings, Royal Asiatic soc. Bengal, 1899, p. 199.

A number of papers on remarkable noises will be found in the Monthly Weather Review, Washington, vols. **23, 26, 31, 35,** between 1895 and 1907.

(53) **Sewell, C. J. T.,** *in* Phil. trans., 1911, **A210.**

(54) **Duhem,** *in* Annales de Toulouse, 1901–1904, (2), 3–5.

(55) **Le Roy,** *in* Comptes rendus, Paris, 1913, Ap. 21, 28, Je. 2; 1914, Ap. 27.

(56) This was first remarked by Kirchoff. *See* **Lamb,** Hydrodynamics, 2d ed., Cambridge, 1895, p. 587.

(57) Report of the U. S. Lighthouse Board, 1882, p. 163. In the Report of the British Association for the Advancement of Science for 1861, Prof. Hennessy discussed the subject and referred to some experiments made by Colladon on the Lake of Geneva in 1826.

(58) For the history of the subject, see: **Beazeley, A.** On phonic coast fog signals. Proc. Instit. civil engin., 1870–71, **32**, pt. 2.

(59) *See* **Henry's** Researches, p. 481, 489, 493, 497, 536, 546, 549; **Tyndall,** Sound, p. 298.

Johnson, A. B., *in* Bull., Phil. soc., Washington, 1881, **4**, p. 135; ibid., 1883, **5**, p. 23.

Livermore, Report, 1894; Nature, London, 1895, Aug. 8, p. 347; Knowledge, London, 1901, June.

(60) According to Price Edwards (Nature, London, 1902, May 29) the silent areas do not occur frequently. Mr. Della Torre suggested to me that they were due to the breaking up of the sound by local eddies or whirlwinds.

(61) **Tyndall,** *in* Proc. Royal soc., London, 1882, **34**, p. 18.

(62) **Fowle, F. E.,** *in* Nature, London, 1895, Nov. 7.

(63) **Rayleigh,** *in* Proc. Royal instit., London, 1902, **17**, p. 1; Nature, London, 1902, **66**, p. 42; "Scientific papers," **5**, p. 1.

(64) Tyndall gives a diagram of this siren trumpet in his "Sound." Numerous suggestions with regard to the most suitable pitch and location of a fog signal are given in Henry's Researches, in Livermore's report, and in the article "Lighthouses" in the Encyclopedia Britannica.

(65) **Henry,** Researches, p. 480; **Ribière,** International maritime congress, London, 1893.

(66) Encyclopedia Britannica, article "Lighthouses."

(67) **Rayleigh,** Scientific Papers, **5**, 1. A mushroom-shaped trumpet is often used on lightships.

(68) An account of these experiments on the Isle of Wight is given in Nature, London, 1902, May 29.

(69) **Flammarion.** The Atmosphere. London, 1873, p. 175.

(70) **Abbe, Cleveland.** Report, *in* Bull. Phil. soc., Washington, 1877, p. 139; **Peck, H. A.,** *in* ibid., 1907, **35**, p. 447.

(71) The possibility of interference was pointed out by E. Mach & B. Doss, and was regarded by them as an objection to C. Abbe's theory. (*See* Berichte, K. k. Akad. d. Wissensch., math.-physikal. Kl., IIA, Wien, 1893, 102.)

(72) **Boys, C. V.,** *in* Report, Brit. assoc. adv. sci., 1892; Nature, London, **47**, p. 415–420, 440–446.

(73) **Mallock, A.,** *in* Proc. Royal soc., 1907, **A79**, p. 262. Boys's experiments have also been discussed by **Stokes,** Memoir and scientific correspondence, **2**, p. 337–357.

(74) **Mallock, A.,** *in* Proc. Royal soc., London, 1907, **A80**, p. 110.

(75) See, for instance, **G. von Niessl** *in* Berichte, K. k. Akad. d. Wissensch., math.-physikal. Kl., IIA, Wien, 1896, **105**, p. 23, and 1912, **121$_2$**, p. 1883.

In the case of the great meteor of September 23, 1910, O. Michalke heard about a dozen detonations of different strengths, two or three being very loud. Another observer heard a detonation like that of a distant gun followed immediately by a roll in the clouds, the sound appearing to come from an approaching region. The noise was quite different and much louder than that of thunder.

The noises mentioned by J. Burton Cleland and H. L. Richardson in letters to Nature, London, 1908, June 4 and August 27, were perhaps due to a meteor.

551.5 ..71 : 551.54 551.5 , 21.061

THE MECHANICS OF ATMOSPHERIC AIR WITHIN CYCLONES AND ANTICYCLONES.[1]

[Communicated to the International Meteorological Congress at Chicago, August, 1893.]

By Geh. Hofrat Prof. Dr. MAX MÖLLER.

[Dated Herzogl. Techn. Hochschule, Braunschweig, May, 1893.]

A. THE DIRECTION OF ROTATION AND THE DISTRIBUTION OF PRESSURE.

By a cyclone we understand a whirling mass of air whose rotation, in the Northern Hemisphere, is accomplished in a direction opposite to that of the movement of the hands of a watch; by an anticyclone, on the other hand, we mean a whirling mass of air whose rotation, in the Northern Hemisphere, corresponds to the movement of the watch hands.

In the Southern Hemisphere the conditions are reversed. In the center of the cyclone low pressure always prevails; hence the cyclone is often spoken of briefly as a depression. In the center of the anticyclone high atmospheric pressure generally prevails; hence this is often called a high pressure or maximum. However, there are also anticyclones with low pressure in the center; but this occurs only when the diameter of the anticyclone is small relatively to the strength of the wind (compare section 15 hereafter).

In the ultimate analysis the movements of the atmosphere are almost exclusively produced by differences of temperature; they are, however, so affected by the influence of the inertia of moving masses of air and so hindered by friction due to mutual mixing of masses of air that a study of these numerous relations must first be undertaken in detail before we can successfully proceed to the explanation of such complex natural processes.

According to the theoretical investigations of our master, the late Prof. Ferrel, in the field of the discussion of atmospheric whirlwinds, we have to distinguish many kinds of cyclones and anticyclones.

B. THE THREE KINDS OF CYCLONES.

(a) We have to mention first the cyclone with descending air and a cold center as it is presented to us in general in that whirl which surrounds the temperate, and partly also the cold zone, and produces the westerly trade, namely, the strong west winds of the "roaring forties."

Other examples of depressions with descending air currents have also been observed. Thus, for example, on April 13, 1893, two depressions of this kind rested upon western Russia and the Baltic Sea, respectively. The presence of descending air in this depression or cylone was shown first by the great dryness of the air and by the absence of any considerable precipitation, especially by the fact that clear sky prevailed in the region of the strongest winds; second, and most important of all, the circumstance that at more than 20 European stations at that time the northwest and northeast winds showed no inflow into the depression, but moved parallel to the isobars or passed from the region of feeble pressure over to that of higher.

[1] The present paper was prepared for publication in 1901, but publication has been delayed for the reasons stated in the REVIEW for February, 1914, 42: 93.